Axial Mixing in Pipe Displacement

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info@gatellc.com

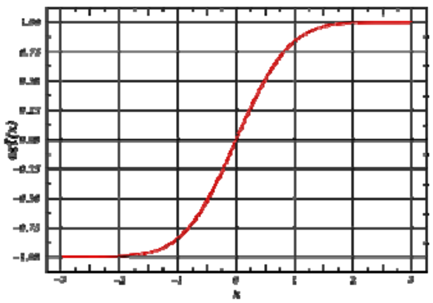

Figure 1: Plot of Error Function

Equations List

$Characteristic\ Length = 2\sqrt{D_l t}$ *(Equation 2)*

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$ *(Equation 3)*

$$\frac{D_l}{vd} = \frac{1}{ReSc} + \frac{ReSc}{192}$$ *(Equation 4)*

$$D_{AB} = \frac{7.4 \times 10^{-12}(\varphi_B M_B)^{1/2} T}{\mu_{AB}\left(V_{M_A} \times 100^3\right)^{0.6}}$$ *(Equation 5)*

$$V_{M_A} = \frac{M_A}{\rho_A}$$ *(Equation 6)*

$$\mathrm{Sc} = \frac{\mu}{\rho D_{AB}}$$ *(Equation 7)*

Association Factor	
Water	2.6
Methanol	1.9
Ethanol	1.5
Nonpolar solvents	1.0

Table 1: Association Factor values for Diffusivity in Liquid Mixtures

It is frequently necessary to displace the contents of a pipeline or umbilical tube (fluid B) with another fluid (fluid A). If we don't use a pig to separate the liquids, there will be mixing at the interface (axial mixing). The mixing zone requires us to overflush the line to effectively remove fluid B from the line.

Below, we address a method of calculating the length of the mixing zone in order to determine the effective overflush requirement for a given pipeline or umbilical tube.

The Taylor Equation—Axial Dispersion in Turbulent Flow

Taylor[1] developed Equation 1 to describe axial dispersion in turbulent flow:

$$C(x,t) = 0.5\left\{1 - erf\left[\frac{x - vt}{2\sqrt{D_l t}}\right]\right\}$$ *(Equation 1)*

The error function (erf) is defined by Equation 3, and is illustrated in Figure 1. The curve illustrates the concentration profile of fluid B across the mixing zone. Inspecting the Taylor equation, we see that the erf is zero when x—vt is zero. This is the time (t) at which the center of the mixing zone arrives at the end of the line. Where the erf equals zero, the Taylor equation gives the mixture concentration of 50% of fluid B.

Characteristic Length, Mixing Zone Length

We can now define the characteristic length as the denominator of the erf, as illustrated in Equation 2.

Per the erf, we see that, with a distance of 1 characteristic length from the center of the mixing zone, the concentration of fluid B is 84.5% at the leading edge or 15.5% at the trailing edge. At a characteristic length of 2, the concentration of fluid B is 99.5% leading edge or 0.5% at the trailing edge. If this is adequate displacement, then we can define the mixing zone length as 4 * characteristic length which is equal to 8 * sqrt (D_l t).

Axial Dispersion Coefficient, D_l

To calculate characteristic length, we need to determine the axial dispersion coefficient, D_l. Richardson and Coulson[2] provide a plot (Figure 2) relating Reynolds Number (Re), Schmidt Number (Sc), and the dimensionless form of dispersion coefficient, D_l/vd. There are several things worth noting on Figure 2.

1) In turbulent flow, D_l is only a function of Re. At Re > 100,000 D_l/vd is fixed.
2) In laminar flow, the Schmidt number dominates.
3) The D_l for gases is relatively low at all values of Re.
4) The highest D_l (worst displacement) will occur at the transition from laminar to turbulent flow.

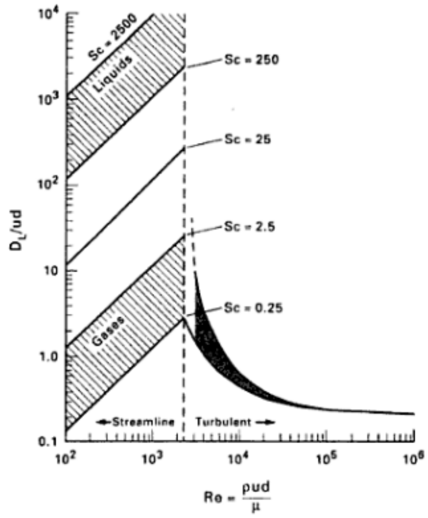

Figure 2: Dispersion Coefficient Plot

Gibson Applied Technology and Engineering, Inc.
16360 Park Ten Place Ste 206 | Houston | TX | 77084 | T 281-398-5781 | F 281-398-0935 | info@gateinc.com

Liquids	T=120°F	
	D_{AB}, m^2/s	Sc
Methanol	5.01×10^{-9}	103
Water	5.13×10^{-9}	110
Decane	1.9×10^{-9}	460
Liquids	**T= 40°F**	
	D_{AB}, m^2/s	Sc
Methanol	2.35×10^{-9}	395
Water	1.6×10^{-9}	969
Decane	8.87×10^{-10}	1792

Table 2: Diffusivity Coefficient and Schmidt Number for component diffusing into similar component

Nomenclature

v	velocity, m/s
D	diameter, m
L	length of pipe, m
X	distance from flow origin point, m
t	time, sec
M	molecular weight, g/mol
T	temperature, K
D_l	axial dispersion coefficient, m^2/s
C(x,t)	concentration of displace fluid
Sc	Schmidt Number
Re	Reynolds Number
μ	viscosity at given temperature, cP
ρ	density at given temperature, g/m^3
D_{AB}	diffusivity coefficient, m^2/s
Φ	association factor
Vm	molar volume, m^3/mol

References

1) Taylor, "The Dispersion of Matter in Turbulent Flow through a Pipe", Proceedings of the Royal Society of London, Vol 223, May 20, 1954

2) Coulson, Richardson, *Chemical Engineering*, Elsevier, 1991

Schmidt Number Estimation

In laminar flow, the Sc dominates. Sc is described via Equation 7. Sc can be calculated via Equation 7, if the diffusivity is known. Diffusivity can be estimated via Equation 5. A simpler approach, which has proven effective, is to estimate Sc based on viscosity alone. Note that Sc is a function of viscosity squared because viscosity is in both the Sc equation and the diffusivity equation.

The following values of Sc should be used for the fluids described below:

- For low viscosity chemicals, such as methanol, use Sc = 250.
- For water, use Sc = 1,000.
- For higher viscosity chemicals, such as MEG, use Sc = 2,500.

Sample Calculation 1

A 10 inch ID (0.254 m) 20 km gas export line is filled with nitrogen and is to be displaced with natural gas. Displacement velocity is 8 m/sec, the Re number is greater than 1,000,000. What is the length of the mixing zone? From Figure 2, D_l/vd = 0.2, hence:

$$D_l = (Fig\ 2\ value) \times v \times d = 0.2 \times 8\ m/sec \times 0.254\ m = 0.406\ m^2/sec$$

$$t = \frac{v}{d} = \frac{20{,}000\ m}{8\ m/sec} = 2{,}500\ sec$$

$$Mixing\ Zone \approx 8 \times \sqrt{D_l \times t} \approx 8 \times \sqrt{0.406\ m^2/sec \times 2{,}500\ sec} \approx 253\ m$$

This agrees with field experience that gas line displacements have small mixing zones.

Sample Calculation 2

A 4 mile (6439 m) 1" ID (0.025 m) umbilical tube containing storage fluid of EG/Water mixture was displaced with diesel with a viscosity of 4 cP. The maximum displacement rate was 2 gpm. This yields Re = 1,270, Velocity = 0.82 ft/sec = 0.25 m/sec, Displacement time = 25,800 sec.

Assuming a Schmidt Number of 2,000 (for pure MEG we would have used 2,500). From Figure 2, D_l/vd is approximately = 7,000, hence:

$$D_l = (Fig\ 2\ value) \times v \times d = 7000 \times 0.25\ m/sec \times 0.025\ m = 43.75\ m^2/sec$$

$$Mixing\ Zone \approx 8 \times \sqrt{D_l \times t} \approx 8 \times \sqrt{43.75\ m^2/sec \times 25{,}800\ sec} \approx 8{,}500\ m\ (longer\ than\ the\ umbilical!)$$

In this case, it took between 1.5 and 2 times the volume of the umbilical to effectively displace the storage fluid.

Conclusions and Cautions

The following conclusions and cautions can be applied to the method described above:

- The method yields rough estimates only. Opportunities to collect field data to validate the model are rare (If you have data we would like to see it.)
- The Taylor equation was developed for miscible fluids in turbulent flow. The method proposed here extends it to laminar flow. Sample Calculation 2, shown above, confirms successful application to immiscible fluids.
- Two fluids provide different Reynolds numbers. Use the one giving the highest value of D_l.
- These equations do not take elevation effects into account. For fluids with different densities, elevation effects may be important.
- If the displaced fluid adheres to the pipe wall, then the tail may be extended.